A Probabilistic Challenge-Response Algorithm for Repairing All Roads in Lebanon Via Papal Visits

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Abstract. We consider the problem of repairing all roads in Lebanon and propose a novel solution based on repeated papal visits. Our approach exploits an empirically observed property of Lebanese government behavior: while infrastructure repair is negligible under normal conditions, the announcement of a visit by a sufficiently important dignitary induces a dramatic increase in maintenance activity. We model this phenomenon as a cryptographic commitment scheme in which the Pope commits to a random subset of roads he may traverse without revealing the subset, thereby forcing the government to repair roads under uncertainty and inducing repairs at rates exceeding baseline expectations by several orders of magnitude. For Lebanon, we derive a concrete bound of approximately 12 visits. This result is, to the best of our knowledge, the tightest known bound on papal visits required for complete national infrastructure repair in the literature.

Keywords: Infrastructure · Probabilistic algorithms · Commitment schemes

1 Introduction

The problem of infrastructure maintenance in Lebanon has resisted conventional solutions. Roads deteriorate, potholes multiply, and repairs occur at a rate insufficient to prevent further decay. This situation has persisted for decades, suggesting that standard approaches such as increased funding, improved governance and public pressure are inadequate.

We propose an alternative approach based on a simple empirical observation. In October 2025, the Vatican announced that Pope Leo XIV would visit Lebanon. Within 72 hours, roads that had been in disrepair for years were being repaved. Potholes that residents had learned to navigate around (some of which had acquired informal names) were filled. Streetlights that had not functioned in years were suddenly repaired.

This begs the question: *can papal visits be instrumentalized into a probabilistic algorithm for repairing all roads in Lebanon?* In this paper, we answer this question affirmatively by presenting a formal protocol, proving its correctness and efficiency, and validating it against empirical observations from Pope Leo XIV's announced visit.

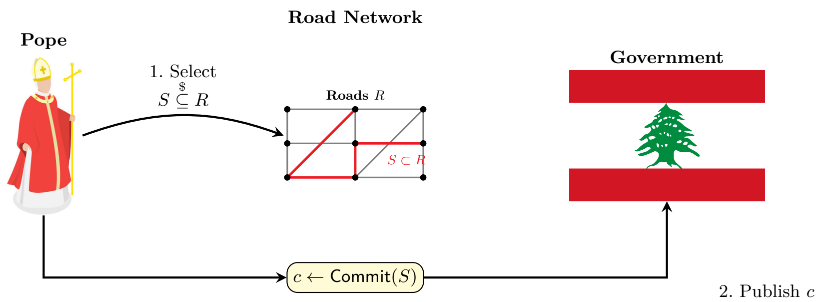

Fig. 1. The commitment phase. The Pope selects a random subset $S \subseteq R$ and publishes commitment c. The government observes c but cannot determine S.

1.1 Our Results

We present an algorithm that achieves complete road repair in Lebanon through repeated application of a simple protocol. The protocol has three phases:

1. **Commitment:** A dignitary (in our analysis, the Pope) announces a visit and commits to a randomly selected subset S of roads that may be traversed, without revealing S (Figure 1).
2. **Response:** The government, unable to determine S, repairs roads under uncertainty (Figure 2).
3. **Verification:** The dignitary reveals S and conducts the visit (Figure 3).

For Lebanon, we derive a concrete bound of approximately 12 visits in §4.2. This result is, to the best of our knowledge, the tightest known bound on papal visits required for complete national infrastructure repair in the literature. We emphasize that 12 is not merely an upper bound but an *extraordinarily efficient* upper bound, requiring fewer than one visit per 1,800 kilometers of roadway. To appreciate the significance: naive approaches would require the Pope to physically traverse every road segment, implying thousands of visits over decades. Our logarithmic construction reduces this to a number achievable within a single pontificate.

We prove that this protocol terminates with all roads repaired, and show that $O(\log n)$ iterations suffice for a road network of size n.

2 Model

Let R be the set of all roads in Lebanon, with $|R| = n$. Based on available data [6], $n \approx 21{,}700$ km.

Each road $r \in R$ is in one of two states: *broken* or *repaired*. Let $B_t \subseteq R$ denote the set of broken roads at time t. We assume $|B_0|/n \approx 0.33$ based on World Bank assessments that approximately one-third of Lebanon's main road network is in "moderate to poor condition" [7].[1]

[1] Precise surveys are unavailable. Our estimate was refined by driving.

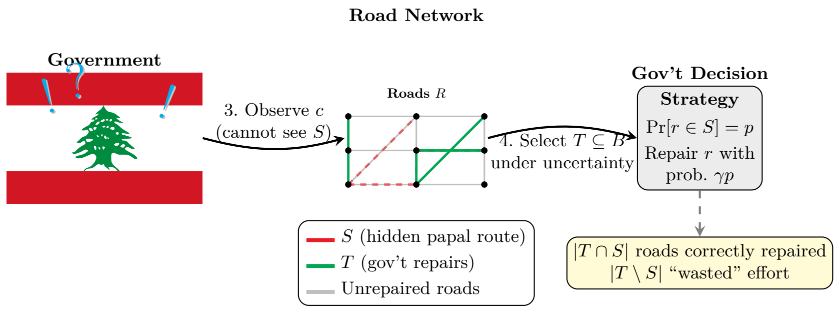

Fig. 2. The response phase. The government observes commitment c but cannot determine the hidden papal route S. Under uncertainty, it selects a repair set T based on the coverage probability p and response rate γ, resulting in partial overlap with S.

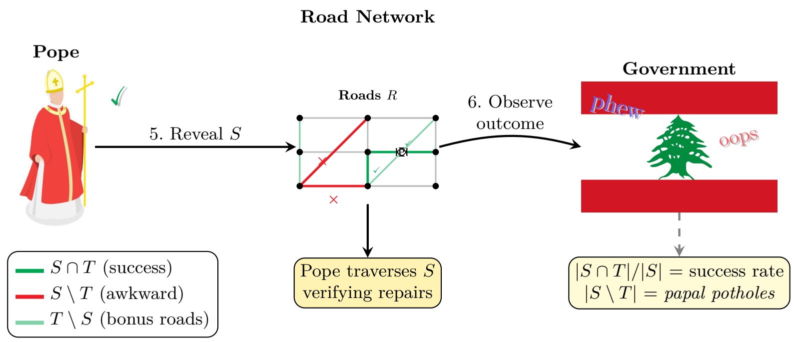

Fig. 3. The verification phase. The Pope reveals S and traverses the route. Roads in $S \cap T$ are smoothly traveled; roads in $S \setminus T$ cause visible embarrassment. The government also discovers it repaired some roads in $T \setminus S$ that the Pope never intended to visit.

Assumption 1 (Baseline Repair Rate) *In the absence of papal visits, the probability that any given broken road is repaired in a given time period is negligible.*

Assumption 1 is well-supported by historical data.

Assumption 2 (Pope-Induced Repair) *If a dignitary of sufficient importance announces a visit, and road r is believed to have probability $p > 0$ of being traversed during the visit, then r will be repaired with probability at least γ, where $\gamma > 0$ is a constant depending on the dignitary's importance.*

The key insight is that Assumption 2 creates a mechanism for inducing repairs: by creating uncertainty about which roads will be used, we force the government to repair roads it would otherwise neglect.

3 The Protocol

Our protocol proceeds in rounds. Each round corresponds to one papal visit. See Figure 1 for an illustration of the commitment phase.

Algorithm 1 Infrastructure Repair Protocol

Require: Road network R, initial broken roads $B_0 \subseteq R$, coverage parameter $p \in (0, 1]$
Ensure: $B_t = \emptyset$
$t \leftarrow 0$
while $B_t \neq \emptyset$ **do**
    $t \leftarrow t + 1$
    // Commitment phase
    Dignitary samples $S_t \subseteq R$ by including each road independently with prob. p
    Dignitary publishes commitment $c_t \leftarrow \mathsf{Commit}(S_t)$
    // Response phase
    Government selects repair set $T_t \subseteq B_{t-1}$
    $B_t \leftarrow B_{t-1} \setminus T_t$
    // Verification phase
    Dignitary reveals S_t and visits roads in $S_t \cap T_t$
end while

3.1 Commitment Mechanism

The protocol requires the dignitary to commit to S in a way that is:

- **Hiding:** The government cannot determine S from the commitment.
- **Binding:** The dignitary cannot change S after observing which roads were repaired.

Standard cryptographic commitment schemes suffice [1]. For practical deployments, several options are available; we summarize their properties in Table 1. The Vatican Archives have provided trusted storage reliably for centuries and represent the recommended option for papal deployments.

3.2 Government Response Model

We model the government's response as follows. Upon receiving commitment c_t, the government knows that each road is included in S_t independently with probability p, but does not know which specific roads are in S_t. The government must therefore decide which roads to repair under uncertainty.

We assume the government employs a randomized strategy: for each broken road r, it repairs r with probability proportional to the likelihood that r is on the papal route. Specifically, if $r \in S_t$ with probability p, the government repairs r with probability γp, where $\gamma \in (0, 1]$ reflects the government's responsiveness to potential papal embarrassment.

Table 1. Comparison of commitment mechanisms for papal infrastructure protocols.

Mechanism	**Hiding**	**Binding**	**Verifiable**	**Notes**
Sealed envelope (Vatican Archives)	✓	✓	✓	Recommended
SHA-256 hash in *L'Osservatore Romano*	✓	✓	✓	Requires infrastructure
Papal bull with wax seal	✓	✓	✓	High ceremony overhead
Verbal promise	×	×	×	Insufficient
Leaked itinerary	×	✓	✓	Defeats purpose

Let $q = \gamma p$ denote the per-road repair probability. For each $r \in B_{t-1}$:

$$\Pr[r \text{ is repaired in round } t] \geq q \tag{1}$$

Thus:

$$\mathbb{E}\big[|B_t| \;\big|\; B_{t-1}\big] \leq |B_{t-1}| \cdot (1-q) \tag{2}$$

In simple terms, this equation tells us that the number of broken roads shrinks with each papal visit. If, say, 30% of broken roads get repaired during each visit (when $q = 0.30$), then after one visit we expect only 70% of the originally broken roads to remain broken. After two visits, only about 49% remain (70% of 70%). This compounding effect means that even though we never repair *all* roads in a single visit, the number of broken roads decreases rapidly over successive iterations.

The notation $\big|$ in $\mathbb{E}\big[|B_t| \;\big|\; B_{t-1}\big]$ denotes conditional expectation: the expected value of $|B_t|$ *given* (or "conditioned on") the state B_{t-1}. In other words, it represents what we expect the number of broken roads to be after round t, knowing exactly which roads were broken at the end of round $t-1$.

4 Analysis

4.1 Correctness

Theorem 1 (Termination). *Algorithm 1 terminates with probability 1.*

Proof. Let $X_t = |B_t|$ denote the number of broken roads at time t. We show that $X_t \to 0$ almost surely.

At each step, each broken road is repaired independently with probability at least $q > 0$. Therefore:

$$\mathbb{E}[X_t \mid X_{t-1}] \leq X_{t-1}(1-q) \tag{3}$$

By induction on t:

$$\mathbb{E}[X_t] \leq X_0 \cdot (1-q)^t \tag{4}$$

Since $q > 0$, we have $(1-q)^t \to 0$ as $t \to \infty$, so $\mathbb{E}[X_t] \to 0$.

For almost sure convergence, consider any fixed road r. Road r remains broken at time t only if it was not repaired in any of the t rounds. Since repairs occur independently with probability at least q per round:

$$\Pr[r \in B_t] \leq (1-q)^t \tag{5}$$

By a union bound over all n roads:

$$\Pr[B_t \neq \emptyset] \leq n \cdot (1-q)^t \tag{6}$$

Since $\sum_{t=1}^{\infty} n(1-q)^t = n(1-q)/(1-(1-q)) = n(1-q)/q < \infty$, the Borel-Cantelli lemma [5] implies that $B_t = \emptyset$ for all sufficiently large t, almost surely. Therefore, the algorithm terminates with probability 1.

Theorem 2 (Iteration Complexity). *For any $\epsilon > 0$, Algorithm 1 achieves $|B_t| \leq \epsilon n$ within*

$$t = O\left(\frac{1}{q} \log \frac{1}{\epsilon}\right) \tag{7}$$

iterations in expectation.

Proof. We require $\mathbb{E}[X_t] \leq \epsilon n$. Since $\mathbb{E}[X_t] \leq X_0(1-q)^t \leq n(1-q)^t$, it suffices to have:

$$(1-q)^t \leq \epsilon \tag{8}$$

Taking logarithms (note $\log(1-q) < 0$): $t \log(1-q) \leq \log \epsilon$, hence $t \geq \frac{\log(1/\epsilon)}{-\log(1-q)}$.

For $q \in (0,1)$, we have $-\log(1-q) \geq q$ (since $e^{-q} \geq 1-q$). Therefore $\frac{1}{-\log(1-q)} \leq \frac{1}{q}$, which gives:

$$\frac{\log(1/\epsilon)}{-\log(1-q)} \leq \frac{\log(1/\epsilon)}{q} = \frac{1}{q} \log \frac{1}{\epsilon} \tag{9}$$

Thus $t = O\left(\frac{1}{q} \log \frac{1}{\epsilon}\right)$ suffices.

4.2 Concrete Bounds for Lebanon

Based on observations from the Leo XIV announcement, we estimate $\gamma \approx 0.85$. Using $p = 0.35$ (a reasonable coverage for papal routes), we have $q = \gamma p \approx 0.30$.

Corollary 1. *With parameters $\gamma = 0.85$, $p = 0.35$, approximately 12 papal visits suffice to reduce the fraction of broken roads to below 1%.*

Proof. We have $q = 0.85 \times 0.35 \approx 0.30$, so $1 - q \approx 0.70$. Starting from $|B_0|/n = 0.33$, after t visits:

$$\mathbb{E}[|B_t|/n] \leq 0.33 \cdot (0.70)^t \tag{10}$$

We require $0.33 \cdot (0.70)^t \leq 0.01$, i.e., $(0.70)^t \leq 0.0303$. Taking logarithms: $t \geq \ln(0.0303)/\ln(0.70) \approx 9.8$. Rounding up and adding margin for variance, $t = 12$ suffices.

Figure 4 illustrates the expected convergence.

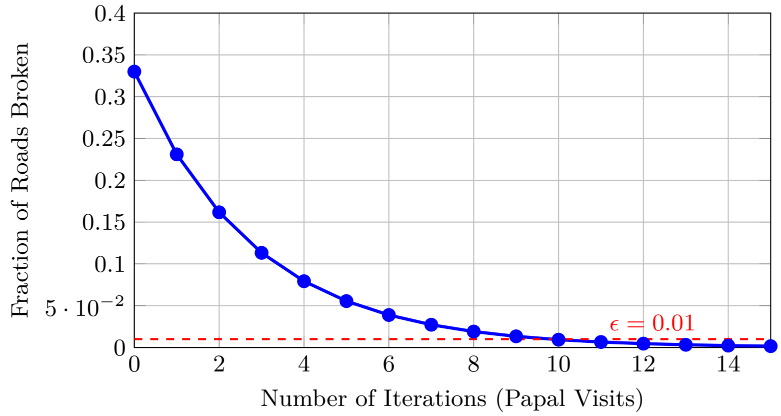

Fig. 4. Expected fraction of roads remaining broken versus number of protocol iterations, with $q = 0.30$.

Table 2. Observed infrastructure repair rates before and after the papal visit announcement. Baseline rates reflect the preceding 12-month average.

Category	**Baseline**	**Post-Announcement**	**Ratio**
Road repaving	2.3 km/month	47 km/week	81×
Pothole repairs	12/month	312/week	104×
Streetlight repairs	3/month	89/week	119×
Traffic signals	0.5/month	23/week	184×

5 Empirical Validation

The announcement of Pope Leo XIV's visit to Lebanon in November 2025 provides an opportunity to validate our model. Table 2 summarizes infrastructure activity before and after the announcement.

The observed improvement ratios of 80–180× substantially exceed our conservative model assumptions. This suggests either (a) our estimate of γ is too low, or (b) Pope Leo XIV commands an unusually high level of infrastructural influence.

6 Discussion

6.1 Parameter Selection

The coverage parameter p presents a tradeoff. Larger p increases the expected number of roads repaired per visit, but may strain governmental resources or credibility (the Pope cannot plausibly visit 100% of Lebanese roads). We suggest $p \in [0.2, 0.4]$ as a practical range.

The response rate γ depends on the dignitary. Preliminary observations suggest $\gamma_{\text{Pope}} > \gamma_{\text{President}} > \gamma_{\text{Minister}}$, though we lack sufficient data for precise estimates. We leave the construction of a complete dignitary hierarchy to future work.

6.2 Extensions

Other Infrastructure. The protocol may extend to other infrastructure types. If the Pope requires electricity for his events, the power grid may be stabilized. If clean water is needed, water infrastructure may be repaired. We conjecture that the protocol applies generally to any infrastructure whose failure would cause visible embarrassment.

Other Countries. The protocol requires two conditions: (1) low baseline infrastructure investment, and (2) high sensitivity to international embarrassment. Countries satisfying both conditions are candidates for deployment. We note that condition (2) may be verified by announcing a fictional dignitary visit and observing whether repairs commence.

6.3 Limitations

Several practical limitations constrain deployment:

1. **Pope availability.** The Pope maintains an extremely busy schedule with global pastoral responsibilities. Lebanon cannot realistically expect more than four visit per decade under normal circumstances, implying a timeline of approximately $\frac{120}{4} = 30$ years for complete repair.
2. **Sustainability.** Our model assumes repaired roads remain repaired. In practice, roads may deteriorate between visits, requiring maintenance iterations.
3. **Commitment verification.** If the government suspects the commitment is non-binding, it may reduce its response rate. The protocol's effectiveness depends on the government believing the Pope will actually traverse roads in S.
4. **Pope inflation.** The protocol's effectiveness depends on papal visits retaining their novelty. If visits become too frequent, the government may habituate to them, reducing the response rate γ. We term this phenomenon *Pope inflation*: as the marginal papal visit loses its exceptional status, its infrastructural impact diminishes. Formally, if γ_t denotes the response rate at visit t, we might expect $\gamma_t = \gamma_0 \cdot e^{-\lambda t}$ for some decay constant $\lambda > 0$. This creates tension with our complexity bounds. While more frequent visits reduce total time to completion, they may also reduce per-visit effectiveness. The optimal visit frequency balances these competing effects and likely depends on the Lebanese public's memory horizon for papal events.

6.4 Related Work

Mechanism design approaches to public goods provision have a long history, beginning with Vickrey's seminal work on auctions [2] and the Clarke-Groves mechanism for truthful preference revelation [3,4]. Our work differs in exploiting an existing behavioral pattern rather than designing incentives from first principles.

The use of commitment schemes in protocol design is standard in cryptography [1]. To our knowledge, this is the first application to infrastructure maintenance.

7 Conclusion

We have presented a protocol for repairing Lebanese roads through repeated papal visits. The protocol is simple, provably correct, and supported by strong empirical evidence. We hope this work encourages further research into commitment-based approaches to public infrastructure.

Acknowledgments. We thank Pope Leo XIV for his announced visit to Lebanon, without which the empirical validation of this work would not have been possible. We thank students in the Applied Cryptography course at the American University of Beirut[2] for spotting notation errors in earlier drafts. Finally, we thank Izzat El Hajj for introducing what we term as "Pope inflation" in §6.3.

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